NONLINEAR PROBLEM

Modified version of bar problem :

$$-\frac{d}{dx}\left(EA\frac{du}{dx}\right) + k_0(1 \pm \alpha u^2)u = f \; ; \; 0 < x < L$$

$$u(0) = 0 \; ; \; EA\frac{du}{dx}\Big|_L = P$$

~ Nonlinear spring model added — HARDENING $\Rightarrow +\alpha$

SOFTENING $\Rightarrow -\alpha$

How to solve this problem ?

~ Weighted residual form :

$$\int_0^L \Big\{ \underbrace{(-EAu')'v}_{\text{integrate by parts}} + k_0(1 \pm \alpha u^2)uv \Big\} dx = \int_0^L f v \, dx$$

$\Downarrow$

$$\boxed{\int_0^L \left(EAu'v' + k_0(1 \pm \alpha u^2)uv\right) dx = \int_0^L f v \, dx + Pv\big|_L}$$

WEAK FORMULATION

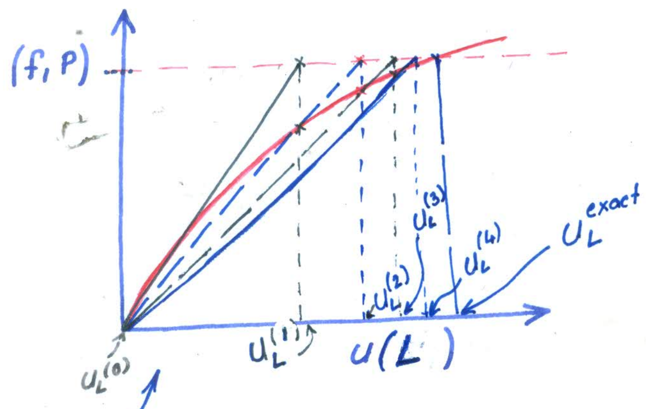
PROBLEM : LHS has terms with $u(x) \leftarrow$ UNKNOWN!

HOW TO SOLVE ?

$\rightarrow$ "Guess" a value for $u(x) \rightarrow u^{(0)}(x)$, put in the LHS and solve for a "better" $u^{(1)}(x)$. Hope that this process terminates when $u^{(i)}(x) \approx u^{(i+1)}(x)$.

$\Downarrow$

ITERATIVE PREDICTOR - CORRECTOR STEPS.



Note that $|U_L^{exact} - U_L^{(i)}| \rightarrow 0$ as "i" increases

PICARD or DIRECT INTEGRATION

$\downarrow$

Given the form before we use:

$$\int_0^L \left(EA\, U_{FE}^{(i+1)\,'} \, \upsilon_{FE}' + k_0 \left(1 \pm \alpha\, U_{FE}^{(i)\,2}\right) U_{FE}^{(i+1)} \upsilon_{FE} \right) dx = \int_0^L f\, \upsilon_{FE}\, dx + P\, \upsilon_{FE}\big|_L$$

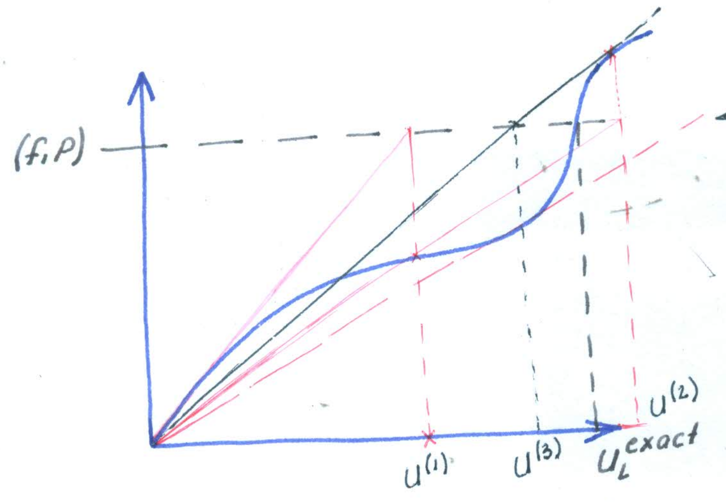
or for $\underline{\upsilon_{FE} = \varphi_e(x)}$, $U_{FE}^{(i+1)} = \sum_{j=1}^{N} d_j^{(i+1)} \varphi_j(x)$

$U_{FE}^{(i)} = \sum_{j=1}^{N} d_j^{(i)} \varphi_j(x) \leftarrow$ KNOWN

$$\sum_{j=1}^{N} d_j^{(i+1)} \int_0^L \left(\varphi_j' \varphi_e' + k_0 \left(1 \pm \alpha\, U_{FE}^{(i)\,2}\right) \varphi_j \varphi_e \right) dx = \int_0^L f \varphi_e\, dx + P\, \delta_{eN}$$

$$\Rightarrow \boxed{[K(U_{FE}^{(i)})]\{d^{(i+1)}\} = \{F^{(i+1)}\}}$$

Solve and check!



- $u^{(2)}$, $u^{(3)}$, $u^{(4)}$ start oscillating about the u^{exact}
- CONVERGENCE DOES NOT HAPPEN

→ STOPPING CRITERION : $\{\Delta d^{(i)}\} = \{d^{(i+1)}\} - \{d^{(i)}\}$

if $\dfrac{|\{\Delta d^{(i)}\}|}{|\{d^{(i+1)}\}|} < \eta$ stop

tolerance ($\sim 10^{-4}$?)

or $\{r^{(i)}\} = \{F^{(i)}\} - [K(u_{FE}^{(i)})]\{d^{(i)}\}$

RESIDUAL

If $\|\{r^{(i+1)}\}\| < \eta$ $\boxed{\text{STOP}}$

$$|\{y\}| = \sqrt{\sum_{i=1}^{N} y_i^2}$$