BENDING DISPLACEMENT

- Point O goes to O'
- Point P goes to P'
- $\tan\theta = \text{slope} = \dfrac{dv}{dx}$

undeformed

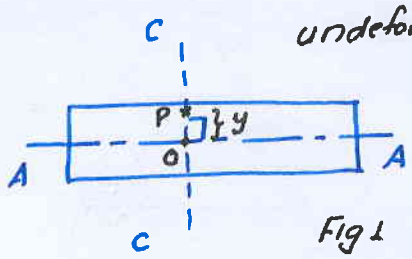


Fig 1

deformed

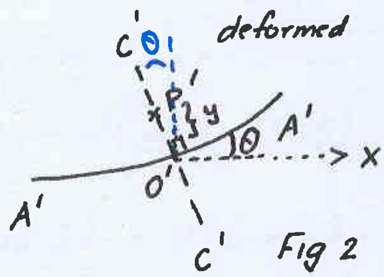
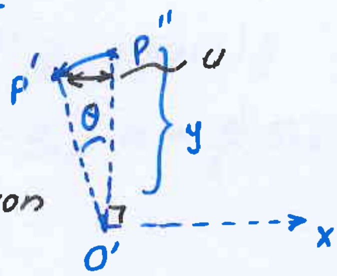


Fig 2

- Point P moves by $P''P'$ in the x-direction



But $P'P'' \approx -y\theta \approx -y\dfrac{dv}{dx}$

$\therefore \quad U_p - U_o \approx -y\dfrac{dv}{dx} \quad \Rightarrow \quad \boxed{U_p \approx U_o - y\dfrac{dv}{dx}}$

$\therefore$ General AXIAL displacement is $U_p(x, y, z) \approx U_o(x,0,0) - y\dfrac{dv}{dx}$

A similar exercise in $\xi - x$ plane gives:

$$U_p(x, y, z) \approx U_o(x,0,0) - \left(\xi\frac{d\bar{v}}{dx}\right)$$

$$\approx \boxed{U_o(x,0,0) - y\frac{dv}{dx} - z\frac{dw}{dx}} \quad (*) \qquad \Downarrow \quad y\underbrace{\frac{d\bar{v}}{dx}\cos\alpha}_{\frac{dv}{dx}} + z\underbrace{\frac{d\bar{v}}{dx}\sin\alpha}_{\frac{dw}{dx}}$$

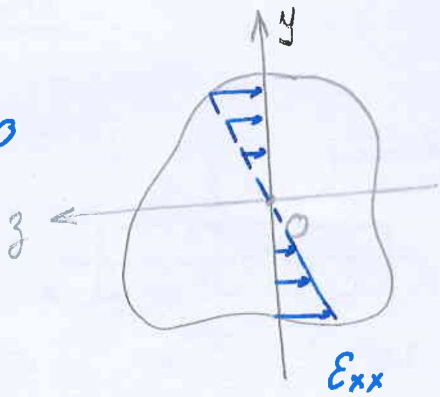
ALTERNATIVE DEFN.

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = -y\frac{d^2v}{dx^2} - z\frac{d^2w}{dx^2}$$

$$\Rightarrow \int\frac{\partial u}{\partial x}dx = \boxed{-y\frac{dv}{dx} - z\frac{dw}{dx}} = \boxed{u(x,y,z) - u_o(x,0,0)}$$

* When $F_{xx} = 0$, $u_o(x,0,0) \equiv 0$ (WHY ??)

General Derivation (alternate form 2):



$\varepsilon_{xx} = 0$ at origin $O \Rightarrow y=0, z=0$

ε_{xx} linear in y, z otherwise

$\Rightarrow \quad \varepsilon_{xx}(x,y,z) \approx a_0(x)\,y + a_1(x)\,z$

$\therefore \quad \sigma_{xx} = E\varepsilon_{xx}\,; \quad \varepsilon_{yy} = -\frac{\nu}{E}\sigma_{xx} = -\nu\varepsilon_{xx}$

$= -\nu(a_0 y + a_1 z)$

But $\varepsilon_{yy} = \frac{\partial v}{\partial y} = -\nu(a_0 y + a_1 z) \Rightarrow v(x,y,z) - v(x,0,0)$

$= -\nu\left(\frac{a_0 y^2}{2} + a_1 yz\right) \Leftarrow$ QUADRATIC IN y, z !!

Retaining terms upto linear gives:

$v(x,y,z) - v(x,0,0) \approx 0$ OR $v(x,y,z) \approx v(x,0,$

$\approx v(x)$

Similarly, $\quad w(x,y,z) \approx w(x) \longleftarrow$ DOES NOT VARY WITH y, z

Now, since $\gamma_{xy} \approx 0 \Rightarrow \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0 \Rightarrow \frac{\partial u}{\partial y} = -\frac{dv}{dx}$

$\Rightarrow \quad u(x,y,z) - u(x,0,0) = \boxed{-y\cdot\frac{dv}{dx} + g(x,z)}$

$\gamma_{xz} \approx 0 \Rightarrow \frac{\partial u}{\partial z} = -\frac{dw}{dx} \Rightarrow \frac{\partial g}{\partial z} = -\frac{dw}{dx}$

$\Rightarrow \quad \boxed{g(x,z) = -z\cdot\frac{dw}{dx}}$

$$\Rightarrow \quad \boxed{\begin{aligned} u(x,y,z) &\approx \underbrace{u(x,0,0)}_{=u_0(x)} - y\frac{dv}{dx} - z\frac{dw}{dx} \\ v(x,y,z) &\approx v(x)\;; \quad w(x,y,z) \approx w(x) \end{aligned}}$$

BENDING DISPLACEMEN[T] FIELD !!