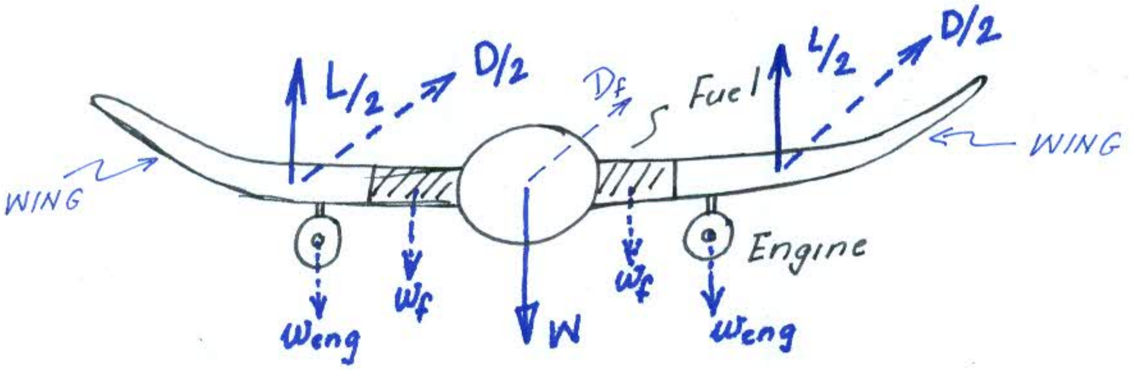
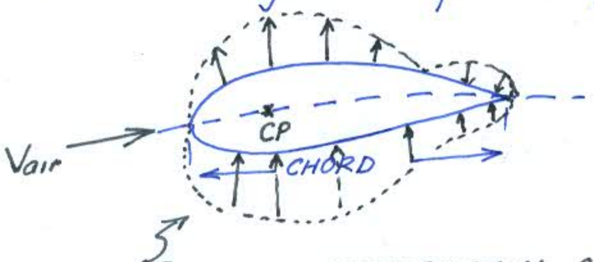
AIRCRAFT + SPACECRAFT

— Lightweight ~ sufficient stiffness + sufficient strength

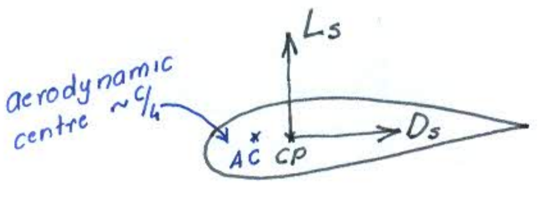


L → Total lift on the aircraft wings
D → Total drag on the aircraft wings
D_f → Drag on fuselage

WING → Primary lift producing structure ~ airfoil shaped cross-section



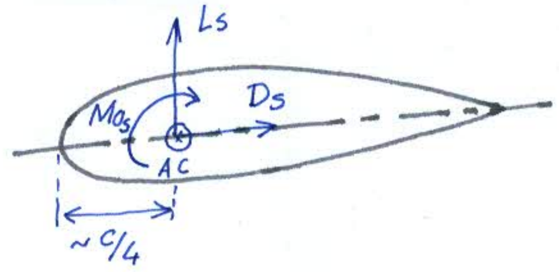
CP ~ centre of pressure
~ point at which resultant lift and drag act



L_s → sectional lift
D_s → sectional drag

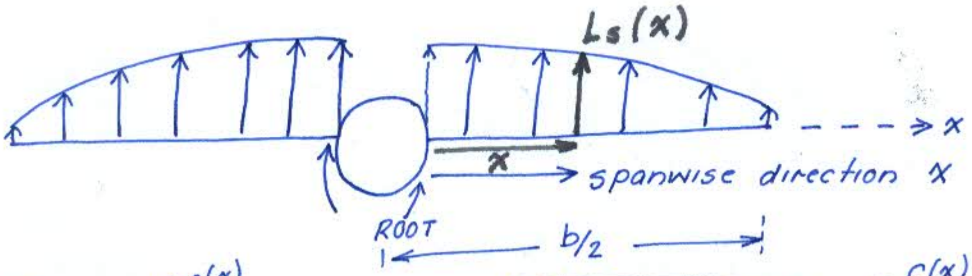
* Note that CP changes as the pressure distribution changes ~ NOT USEFUL from analysis point of view.

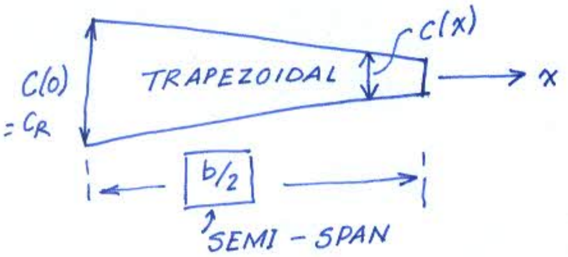
So the AC (aerodynamic centre) is used $\Rightarrow$ at $c/4$ about which L_s, D_s produce a constant moment M_{0s}, for all pressure distributions

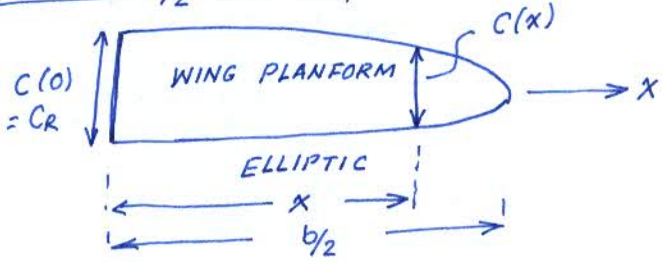


$\Rightarrow$ For all the analysis that we will do, we will use the distributed L_s, D_s, M_{0s}.

How do we obtain the sectional loads?
~ SCHRENK'S FORMULA (based on geometry of planform)







$c(x)$ $\longrightarrow$ mean chord length at location x along the span

ELLIPTIC : $$c(x) = \frac{4S}{\pi b}\sqrt{1 - \left(\frac{2x}{b}\right)^2}$$ [S is wing planform area]

SCHRENK $\rightarrow$ Lift varies the same way as the chord, i.e

$$L_{E_s}(x) = \frac{4L}{\pi b}\sqrt{1 - \left(\frac{2x}{b}\right)^2}$$

TRAPEZOIDAL $$L_{T_s}(x) = L_{T_s}(0)\left(1 - \frac{2x}{b}(1 - \lambda)\right)$$

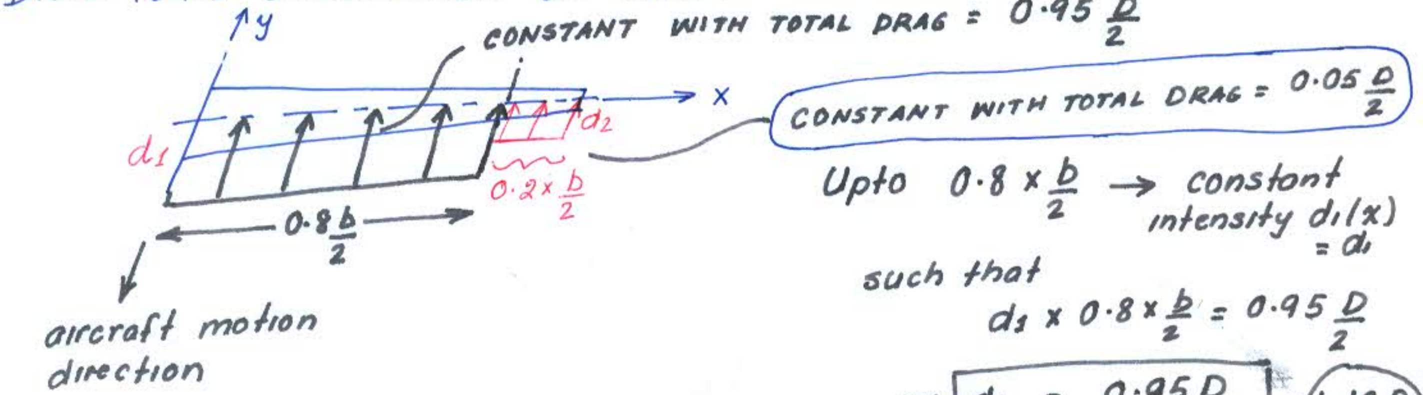
$$\Rightarrow L = \int_{-b/2}^{b/2} L_{T_s}(x)\,dx = L_{T_s}(0)\cdot\frac{b(1+\lambda)}{2} \leftarrow \lambda \rightarrow \text{taper ratio}$$

$$\Rightarrow L_{T_s}(x) = \frac{2L}{b(1+\lambda)}\left(1 - \frac{2x}{b}(1-\lambda)\right)$$

SCHRENK'S FORMULA : The lift for the actual wing planform is the average of the elliptic and trapezoidal lift distributions

$$\boxed{L_s(x) = \frac{1}{2}\left(L_{E_s}(x) + L_{T_s}(x)\right)} \Leftarrow \text{distributed transverse load.}$$

DRAG FORCE DISTRIBUTION ON WING :



Upto $0.8 \times \frac{b}{2} \rightarrow$ constant intensity $d_1(x) = d_1$

such that

$$d_1 \times 0.8 \times \frac{b}{2} = 0.95\,\frac{D}{2}$$

or $\boxed{d_1 = \frac{0.95\,D}{0.8\,b}} \approx 1.19\,\frac{D}{b}$

Similarly, $0.2 \times \frac{b}{2} \times d_2 = 0.05 \times \frac{D}{2}$

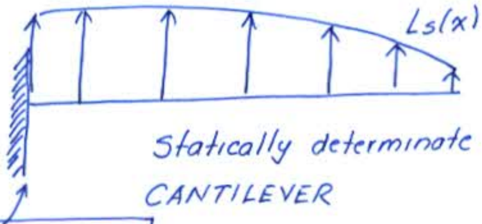
$$\Rightarrow \boxed{d_2 = \frac{0.05\,D}{0.2\,b}} = 0.25\,\frac{D}{b}$$

→ $d_1(x)$, $d_2(x)$ → $D_s(x)$ gives the transverse load distribution in the y-direction

→ When shifted to the Aerodynamic centre, we get $L_s(x)$, $D_s(x)$, $M_{O_s}(x)$ as the sectional load distribution

Given these sectional EXTERNAL distributed loads, we need to find the sectional RESULTANT SHEAR FORCE, BENDING MOMENT & TWISTING MOMENTS →

(BENDING + TORSION of Wing)



Statically determinate CANTILEVER

Root connected to fuselage

SUPPORT IDEALIZATION

$x = 0$ is root, connected to fuselage with end plates which are riveted + a through spar that gives more rigidity

$x = b/2$ is free ⇒ CANTILEVER BEAM

⇒ RIGID SUPPORT AT ROOT

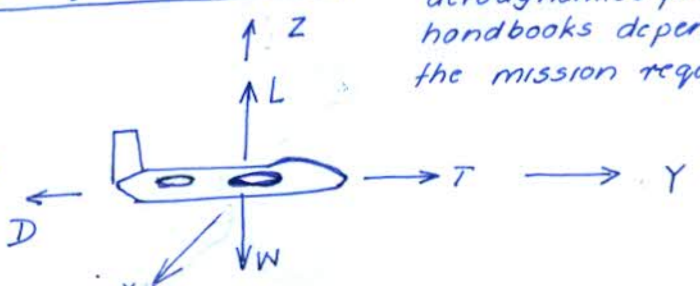
⇒ The resultant shear force $V(x)$; bending moment $M_b(x)$; twisting moment $M_T(x)$ can now be found as:

$$\underset{-V(L)}{V(x)} = \int_{L = b/2}^{x} L_s(x)\, dx \quad ; \quad \underset{-M(L)}{M(x)} = -\int_{L = b/2}^{x} V(x)\, dx \quad ; \quad M_T(x) = \int_{L = b/2}^{x} M_{T_s}(x)\, dx$$

Free-edge conditions: $\boxed{V(L) = M(L) = 0}$

≫ DESIGN NOTES: Who gives L, D? → These come from the aerodynamics people or handbooks depending on the mission requirements.

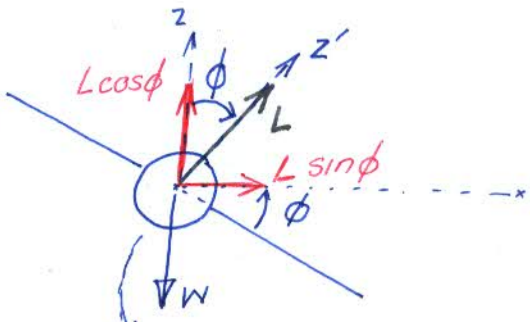
Steady-level flight ⇒



$\boxed{L = W}$ $\boxed{D = T}$

Accelerated Level flight ⇒ $\boxed{L = W}$; $T - D = ma_Y$ ⇒ $\boxed{D = T + ma_Y}$

Level Bank → horizontal circular motion



$$\Rightarrow L\cos\phi = W$$

$$L\sin\phi = \frac{mV^2}{R}$$

speed of aircraft (V); mass of aircraft (m); bank radius (R)

$$\Rightarrow L\sqrt{1-\cos^2\phi} = \frac{mV^2}{R} = L\sqrt{1-(W/L)^2}$$

$$\Rightarrow \frac{V^2}{Rg} = \frac{L}{W}\sqrt{1-(W/L)^2}$$

$$\Rightarrow \sqrt{(L/W)^2 - 1} = \boxed{\frac{V^2}{Rg} = \sqrt{n^2-1}}$$

Note that as R decreases (tighter turn) and as V increases, $\frac{V^2}{Rg}$ increases $\Rightarrow$ n has to be higher

n → LOAD FACTOR

<u>Note</u>:

$n = 1$ for steady-level flight

$n > 1$ for banking flight

$n > 1$ for pull-out of a dive

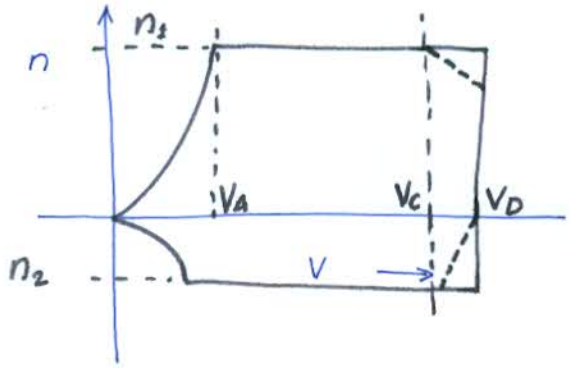
Obviously, a vehicle cannot be made to do any maneuver, as tighter maneuver at higher speeds means higher n and hence HIGHER LIFT requirement ⇒ MORE LOAD ON WINGS ~ FAILURE

Restrictions on maneuvers from structural integrity point of view

FAR – Federal Aviation Regulations
BCAR, DGCA
} CIVILIAN AIRCRAFTS

MIL, CEMILAC — Military standards

What do they give? V - n diagrams (V: speed; n: load factor)



$n_1 \approx 3$ for civilian
$\approx 6-9$ for military

$n_2 \approx -1$ for civilian
$\approx -(2-3)$ for military

$V_A \rightarrow$ maximum stall speed; $V_C \rightarrow$ design cruise speed;
$V_D \rightarrow$ design dive speed

⇛ Normally $-1 \leq n \leq 3$ (LIMIT LOAD FACTOR) or $-W \leq L \leq 3W$ are loads that the vehicle can encounter during its full flight regime (FLIGHT ENVELOPE)

STRUCTURAL DESIGN → need factor of safety (FS) for inadvertent excursions beyond the limit loads ~ $FS = \begin{cases} 1.5 \sim \text{manned} \\ 1.25 \sim \text{unmanned} \end{cases}$

Ultimate load factor $n_u = FS \times n_L \Rightarrow \boxed{n_u \approx 4.5}$

$$\Rightarrow \boxed{L^{design}_{Ultimate} = 4.5\, W_{aircraft}}$$ ← at this load neither **YIELDING** nor **BUCKLING** should happen anywhere

Note: $W_{wing} \approx 0.1 \sim 0.2\, W_{aircraft}$

$$\Rightarrow \frac{L}{W_{wing}} \approx \frac{4.5}{0.1} \approx 45 \text{ or } 23$$

NEED GOOD MATERIAL!

or $\boxed{\text{Wing carries almost } 25 \sim 50 \text{ times its own weight}}$