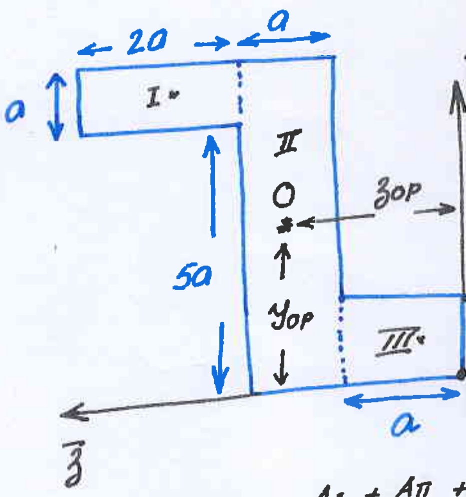
Ex 1 : UNSYMMETRIC SECTION (Z).



Finding centroid :

$$y_{OP} = \frac{1}{A}\int_A \bar{y}\, dA \quad ; \quad z_{OP} = \frac{1}{A}\int_A \bar{z}\, dA$$

Axes chosen arbitrarily

$$A = A_I + A_{II} + A_{III} = 2a^2 + 6a^2 + a^2 = 9a^2$$

$$A \cdot y_{OP} = \bar{y}_I A_I + \bar{y}_{II} A_{II} + \bar{y}_{III} A_{III}$$
$$= (5.5a)\,2a^2 + (3a)\,6a^2 + (0.5a)\,a^2$$
$$= (11 + 18 + 0.5)\,a^3 = 29.5a^3$$

$$\Rightarrow \boxed{y_{OP}} = \frac{29.5a^3}{9a^2} = \boxed{3.277a}$$

$$A\, z_{OP} = \bar{z}_I A_I + \bar{z}_{II} A_{II} + \bar{z}_{III} A_{III}$$
$$= (3a)\,2a^2 + (1.5a)\,6a^2 + (0.5a)\,a^2$$
$$= (6 + 9 + 0.5)\,a^3 = 15.5a^3 \Rightarrow \boxed{z_{OP}} = \frac{15.5}{9}a = \boxed{1.722a}$$

Finding $I_{\bar{z}\bar{z}}|_O = \int_A y^2\, dA \Rightarrow$ obtain it piecewise!

$$I_{zz}|_O = I^{I}_{zz}|_O + I^{II}_{zz}|_O + I^{III}_{zz}|_O$$

(Use // axis theorem) : $I^{k}_{zz}|_O = I^{k}_{zz}|_{C_k} + (y_{C_k})^2 A_k$

where $C_k \rightarrow$ centroid of area A_k.

$$I^{I}_{zz}|_O = \frac{1}{12}\cdot \times 2a \times a^3 + (5.5a - 3.277a)^2 \times 2a^2$$

$$I^{II}_{zz}|_O = \frac{1}{12} \times a \times (6a)^3 + (3a - 3.277a)^2 \times 6a^2$$

$$I^{III}_{zz}|_O = \frac{1}{12} \times a \times a^3 + (0.5a - 3.277a)^2 \times a^2$$

* Work the algebra through.

$$I_{YZ}\big|_O = I_{YZ}^{I}\big|_O + I_{YZ}^{II}\big|_O + I_{YZ}^{III}\big|_O$$

with $$I_{YZ}^{k}\big|_O = I_{YZ}^{k}\big|_{C_k} + (y_{C_k})\cdot(z_{C_k})\,A_k$$

$$\therefore\quad I_{YZ}^{I}\big|_O = 0 + (5.5a - 3.277a)(3a - 1.722a)\times 2a^2$$

$$I_{YZ}^{II}\big|_O = 0 + (3a - 3.277a)(1.5a - 1.722a)\times 6a^2$$

$$I_{YZ}^{III}\big|_O = 0 + (0.5a - 3.277a)(0.5a - 1.722a)\times a^2$$

$$I_{YY}\big|_O = I_{YY}^{I}\big|_O + I_{YY}^{II}\big|_O + I_{YY}^{III}\big|_O$$

$$I_{YY}^{k}\big|_O = I_{YY}^{k}\big|_{C_k} + (z_{C_k})^2 A_k$$

$$\therefore\quad I_{YY}^{I}\big|_O = \frac{1}{12}\,a\times(2a)^3 + (3a - 1.722a)^2\times 2a^2$$

$$I_{YY}^{II}\big|_O = \frac{1}{12}\times 6a\times(a)^3 + (1.5a - 1.722a)^2\times 6a^2$$

$$I_{YY}^{III}\big|_O = \frac{1}{12}\times a\times a^3 + (0.5a - 1.722a)^2\times a^2$$

* Now that we have $I_{YY}\big|_O$, $I_{YZ}\big|_O$, $I_{ZZ}\big|_O$ given any M_y, M_z we can get $\frac{d^2v}{dx^2}$, $\frac{d^2w}{dx^2}$, $\sigma_{xx}(x,y,z)$ at any point in the cross-section.