Finite Element Method (AE675)

Why do we compute?

- To "solve" real-life problems – given in terms of their mathematical definitions
  – PHYSICS of things in the language of MATHEMATICS
- Solve real life problems to understand whats happening and to DESIGN.
- ANALYZE effect of changes – WHAT IF MOMENTS!

How do we compute?

- MODEL reality using mathematical laws in terms of FIELD variables (e.g. displacement, temperature, velocity, electric displacement, etc).
  – HOW GOOD IS THE MODEL?
  Counter question ~ ⟨WITH RESPECT TO WHAT?⟩
  – IDEALIZATION – removal of "unnecessary" complexity to get simplified mathematical equations (linear, static, small perturbation, ...)
  – Global conservation laws (of mass, momentum, energy, charge, ...)
- Convert mathematical conservation laws into IBVP's (initial-boundary value problems) ~ PARTIAL DIFFERENTIAL EQUATIONS.
- Convert IBVP's into forms that a COMPUTER can handle ~ MATRIX PROBLEM

$$[K]\{x\} = \{F\}$$

WHERE DO WE COMPUTE ?

- All disciplines — engineering, biology, chemistry, physics, geo-tech, etc.

WHAT do we compute ?

- Compute to obtain field variables that define the state of the system (as we understand it!)

- Will it break ? — stress, strain, fracture parameters
  Will it melt ? — Temperature, heat flux
  Will it choke ? — Flow rate, pressure

* ⟨I⟩ IDEALIZED MODEL OF REALITY ⟨R⟩

MODELING ERROR

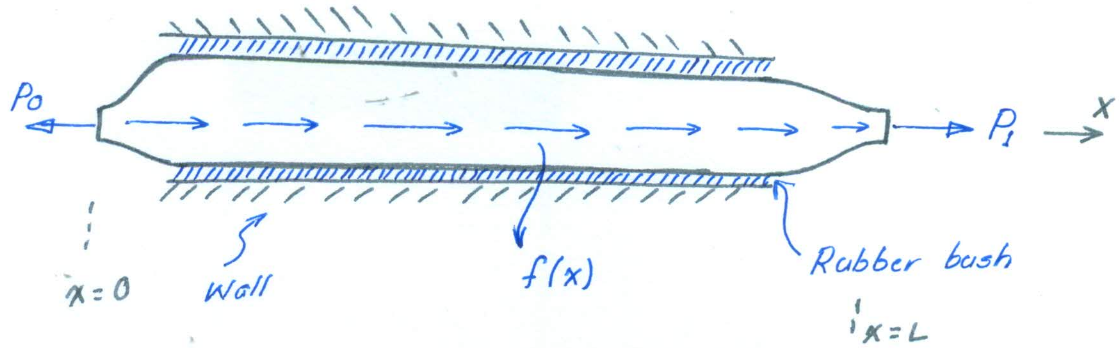
* ⟨I_h⟩ APPROXIMATE solutions of the mathematical problems

DISCRETISATION / APPROXIMATION ERROR

Questions:

- Does it converge to EXACT solution (of idealized model)? How close is I_h to I ?
- How fast does it converge ?
- What is good enough for me ?

EXAMPLE:



Reality: A metal rod of cross-sectional area $A(x)$, with a snugly fit rubber bush, is enclosed in a container and is subjected to a distributed axial load $f(x)$ and end-loads P_0, P_1 as shown.

also idealizations

Idealized model: * Rod ~ linear elasticity, only axial stress

i.e. $\sigma_{xx} = E\,\varepsilon_{xx} = E\frac{du}{dx}$

* Rubber bush ~ linear spring (distributed) with constant $k(x)$. ⟨N/m⟩

* Small deformation, slender member

⇓

1D equation instead of 3D elasticity

$$\boxed{\frac{d}{dx}\left(EA\frac{du}{dx}\right) + \underset{-k(x)u(x)}{f(x)} = 0 \quad , \quad 0 < x < L} \quad \text{— (A)} \leftarrow \text{governing eqn.}$$

with $F_{xx}(x) = \sigma_{xx}(x)\,A(x) = EA\frac{du}{dx}$ ← AXIAL FORCE

Also, $$\boxed{F_{xx}\big|_L = P_1 \;;\; F_{xx}\big|_0 = P_0} \Leftarrow \text{Boundary Condition — (B)}$$

Concept of residual:

Let $r(x) = \frac{d}{dx}\left(EA\frac{du}{dx}\right) - ku + f$ be the residual or the <u>RESULTANT FORCE</u> on an infinitesimal piece of size Δx.