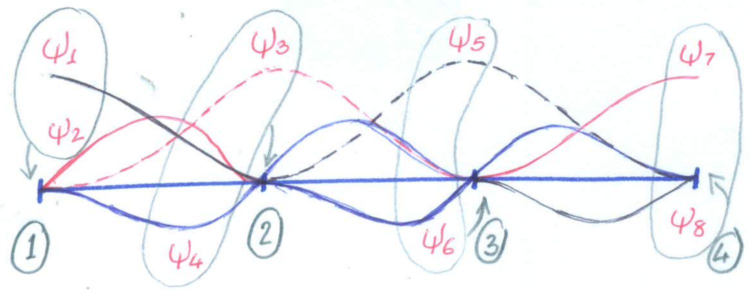
HERMITE (C^1) BASIS FUNCTIONS

We will now need to enforce continuity of displacement $v_h(x)$ and $\frac{dv_h}{dx}(x)$ at the nodes. The basis functions are constructed as:



Each node I has $\underline{\underline{2}}$ basis functions attached as: ψ_{2I-1} and ψ_{2I} such that: (In element I_I)

$$\underline{\psi_{2I-1}(X_I) = 1} \quad ; \quad \frac{d}{dx}\psi_{2I-1}\Big|_{X_I} = 0 \quad ; \quad \psi_{2I-1}\Big|_{X_{I+1}} = 0 \quad ;$$

$$\frac{d}{dx}\psi_{2I-1}\Big|_{X_{I+1}} = 0$$

And $$\psi_{2I}(X_I) = 0 \; ; \; \underline{\frac{d\psi_{2I}}{dx}\Big|_{X_I} = 1} \; ; \; \psi_{2I}(X_{I+1}) = 0 \; ; \; \frac{d\psi_{2I}}{dx}\Big|_{X_{I+1}} = 0$$

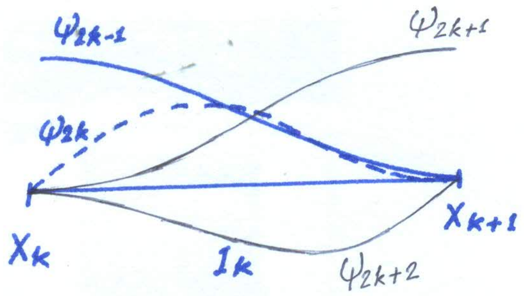
In element I_{I-1}:

$$\psi_{2I-1}(X_{I-1}) = 0 \; ; \; \frac{d}{dx}\psi_{2I-1}(X_{I-1}) = 0$$

$$\psi_{2I}(X_{I-1}) = 0 \; ; \; \frac{d}{dx}\psi_{2I}\Big|_{X_{I-1}} = 0$$

* Note that ψ_{2I-1}, ψ_{2I} are non-zero in elements I_{I-1}, I_I only.

As before, note that in EACH element, the non-zero ψ_i's are



Local numbering:

$$N_1^{H,k}(x) = \psi_{2k-1}\Big|_{I_k} \quad ; \quad N_2^{H,k}(x) = \psi_{2k}\Big|_{I_k} \quad ;$$

$$N_3^{H,k}(x) = \psi_{2k+1}\Big|_{I_k} \quad ; \quad N_4^{H,k} = \psi_{2k+2}\Big|_{I_k}$$

* Note that such a definition of basis functions allows for continuity of value and derivative, at each node.

* Each ψ_i (or $N_\ell^{H,k}$) satisfies 4 conditions in each element. $N_\ell^{H,k}$ is thus a polynomial of degree 3 (4 constants define a cubic) – hence called HERMITE cubic polynomials or basis functions. Example:

Let $N_1^{H,k}(x) = a_0 + a_1(x-x_k) + a_2(x-x_k)^2 + a_3(x-x_k)^3$

$N_1^{H,k}(x_k) = 1 = a_0$

$\dfrac{dN_1^{H,k}}{dx}(x_k) = 0 \Rightarrow a_1 = 0$

$N_1^{H,k}(x_{k+1}) = 0 \Rightarrow 1 + a_2 h_k^2 + a_3 h_k^3 = 0 \quad — (*)$

$\dfrac{dN_1^{H,k}}{dx}(x_{k+1}) = 0 \Rightarrow 2a_2 h_k + 3a_3 h_k^2 = 0$

$\Rightarrow a_2 = -\frac{3}{2} a_3 \cdot h_k$

From $(*)$: $1 - \frac{3}{2} a_3 h_k^3 + a_3 h_k^3 = 0 \Rightarrow a_3 = \dfrac{2}{h_k^3}$

$$\Rightarrow \boxed{N_1^{H,k}(x) = 1 - 3\frac{(x-x_k)^2}{h_k^2} + 2\frac{(x-x_k)^3}{h_k^3}}$$